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SIX DIMENSIONAL BIANCHI TYPE-I COSMOLOGICAL MODELS IN $f(R,T)$ GRAVITY

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ABSTRACT
In this paper, we have obtained two six dimensional exact solutions of Bianchi type-I space-time in the framework of $f(R,T)$ theory of gravity using assumption of constant deceleration parameter and variation law of Hubble parameter. These two solutions correspond to two different cosmological models of the universe. We also discuss the physical behavior of both the models.

Keywords : $f(R,T)$ theory of gravity, six dimensional Bianchi type-I space-time, exact solutions. Cosmological models.

INTRODUCTION
Einstein' general relativity is a successful theory to explain most of the known gravitational phenomena but it fails to explain the accelerating expansion of the universe. It is now proved from observational and theoretical fact that the universe is not only expanding but also accelerating. In order to explain the nature of the accelerated expansion, a number of theoretical models have been proposed by the authors. Justification of the current expansion of the universe comes from modified or alternative theories of gravity. The f(R) gravity, f(T) gravity and f(R,T) theory of gravity are such examples of modified gravity theories.

f(R) theory of gravity is the modification of general theory of relativity proposed by Einstein. This theory plays an important role in describing the evolution of the universe. Many authors have investigated f(R) gravity in different contest. Carrol et al [1] explained The presence of late time cosmic acceleration of the universe in f(R) gravity. Nojiri and Oditsov [2,3] proved that the f(R) theory of gravity provides very natural unification of the early time inflation and late time acceleration. Multamaki and Vilja [4,5] investigated vacuum and perfect fluid solutions of spherically symmetric space time in f(R) gravity. Hollenstein l. et al [6] , M. Shrif et al [7] explored cylindrically symmetric vacuum and non vacuum solutions in f(R) theory of gravity. Adhav K. S. [8] discussed the Kantawski- Sachs string cosmological model in f(R) theory of gravity. Reddy et al [9] studied vacuum solutions of Bianchi type – I and V models in f(R) theory with a special form of deceleration parameter. Ladke L. S. [10] studied the Bianchi type –I (Kasner form) cosmological model in f(R) theory of gravity,
Another recently developed theory is the f(T) theory of gravity in which Weitzenbock connection is used instead of Levi- Civita connection. Yang R. J.[11] discussed what constraint of the coupling term may be put in f(T) theories from observations of the solar systems. M. Sharif et al [12] considered spatially homogeneous and anisotropic Bianchi type–I universe in $f(T)$ gravity theory. Wei H. et al [13] tried to constrain $f(T)$ theories with the fine structure constant. Bamba K. et al [14] studied the cosmological evolution of the equation of state for dark energy with the combination of exponential, logarithmic and $f(T)$ theories. Ratbay M. [15] has shown that the acceleration of the universe can be understood by $f(T)$ gravity models.
Harko et. al. [16] proposed a new generalized theory know as $f(R,T)$ theory of gravity. Gravitational Lagrangian involves the arbitrary function of the scalar Curvature R and the trace of the energy momentum tensor$T$. Houndjo [17] reconstructed $f(R,T)$ gravity by taking $f(R,T) = f_1(R,T) + f_2(R,T)$. Adhav K.S. [18] studied the exact solution of $f(R,T)$ field equations for locally rotationally symmetric Bianchi type-I space time. M. Farasat Shamir et al [19] obtained the exact solutions of Bianchi types-I & V models in $f(R,T)$ by using the assumption of constant deceleration parameter and variation of law of Hubble parameter. Reddy D. R. K. [20] discussed the LRS Bianchi type-II universe in $f(R,T)$ theory.

M. Sharif et al [21] discussed a non equilibrium picture of thermodynamics at the apparent horizon of FRW universe.
The study of higher dimensional physics is important because of several results obtained in the development of super string theory. The study of higher dimensional space- time provides an idea that our universe is much smaller at early stage of evolution as observed today. Weinberg [22] studied the unification of fundamental forces with gravity, which reveals that the space- time should be different from four. Kaluza and Klein [23, 24] have done remarkable work by introducing an idea of higher dimension space-time. Many researcher inspired to entered in to the field of higher dimension theory to explore knowledge of universe. Wesson [25 ,26] and D.R. K. Reddy {27] have studied several aspects of five dimensional space-time in variable mass theory and bi-metric theory of relativity respectively. Lorentz and Petzold [28], Ibanez and Verdaguer [29] , ,Adhav K. S.et al [30] have studied the multidimensional cosmological models in general relativity and in other alternative theories of gravitations..
Inspiring with the above research work we have obtained six dimensional exact solutions of Bianchi type-I space-time assuming constant deceleration parameter and variation law of Hubble parameter proposed by Berman et al. [31] which corresponds to two different cosmological models. The first solution corresponds to singular model and second one to non-singular model. Physical aspects of derived models are also discussed

SIX DIMENSIONAL FIELD EQUATIONSiIN $f(R,T)$ GRAVITY

Six dimensional field equations in $f(R,T)$ theory of gravity are given by

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T)g_{ij} - (\nabla_i \nabla_j - g_{ij}\Box) f_R(R,T) = kT_{ij} - f_T(R,T)(T_{ij} + \theta_{ij}) \; (i, j = 1,2,...,6),$$
(1)

where $f_R(R,T) \equiv \frac{\partial f_R(R,T)}{\partial R}$, $f_T(R,T) \equiv \frac{\partial f_T(R,T)}{\partial T}$, $T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\partial\left(\sqrt{-g}\; L_m\right)}{\partial g^{ij}}$, $\theta_{ij} = -pg_{ij} - 2T_{ij}$

$\Box \equiv \nabla^i \nabla_i$, ∇_i is the covariant derivative.
The energy momentum tensor for perfect fluid yields

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \quad ,$$
(2)

where ρ and p are energy density and pressure of the fluid respectively .
Contracting the above field equations (2), we have

$$f_R(R,T)R + 5\Box f_R(R,T) - 3f(R,T) = kT - f_T(R,T)(T + \theta),$$
(3)

Also above field equations (i), can be written as

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T)g_{ij} - (\nabla_i \nabla_j - g_{ij}\Box) f_R(R,T) = kT_{ij} + f_T(R,T)(T_{ij} + pg_{ij}),$$
(4)

Harko et. al. [16] gives three class of models out of which we used $f(R,T) = R + 2f(T)$ for this models equation (5) can be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = kT_{ij} + 2f'(T)T_{ij} + [f(T) + 2p\, f'(T)]g_{ij},$$
(5)

where overhead prime denotes derivative w.r.to. *T*.
We also choose

$$f(T) = \lambda T \; ,$$ where λ is constant.
(6)

EXACT SOLUTIONS OF BIANCHI TYPE-I SPACE- TIME IN V_6

In this section we find exact solutions of five dimensional Bianchi type-I space time in $f(R)$ theory of gravity. The line element of Bianchi type-I space-time in V_5 is given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 (dy^2) - C^2 (dz^2 + du^2 + dv^2),$$
(7)

where A, B and C are functions of t only.

The corresponding Ricci scalar is

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 3\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + 3\frac{\dot{B}\dot{C}}{BC} + 3\frac{\dot{A}\dot{C}}{AC} + 3\frac{\dot{C}^2}{C^2}\right], \tag{8}$$

where overhead dot means derivative with respect to t.

From equation (5), we obtained

$$\frac{\dot{A}\dot{B}}{AB} + 3\frac{\dot{B}\dot{C}}{BC} + 3\frac{\dot{A}\dot{C}}{AC} + \frac{3\dot{C}^2}{C^2} = (12\pi + 3\lambda)\rho - 3\lambda p, \tag{9}$$

$$\frac{\ddot{B}}{B} + 3\frac{\ddot{C}}{C} + 3\frac{\dot{B}\dot{C}}{BC} + \frac{3\dot{C}^2}{C^2} = \lambda\rho - (12\pi + 5\lambda)p, \tag{10}$$

$$\frac{\ddot{A}}{A} + 3\frac{\ddot{C}}{C} + 3\frac{\dot{A}\dot{C}}{AC} + \frac{3\dot{C}^2}{C^2} = \lambda\rho - (12\pi + 5\lambda)p, \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + 2\frac{\dot{C}\dot{A}}{CA} + \frac{\dot{C}}{C^2} = \lambda\rho - (12\pi + 5\lambda)p, \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + 2\frac{\dot{C}\dot{A}}{CA} + \frac{\dot{C}}{C^2} = \lambda\rho - (12\pi + 5\lambda)p \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + 2\frac{\dot{C}\dot{A}}{CA} + \frac{\dot{C}}{C^2} = \lambda\rho - (12\pi + 5\lambda)p \tag{14}$$

The left hand side of equation (12), (13) and (14) are identical because of the metric function *C* is common along *z* , *u and v* directions.

The system of these four non-linear differential equations consist of five undefined functions i.e. *A*,*B*,*C*, p and ρ.

we consider well known relation between Hubble parameter *H* and average scale factor" *a* "given as

$$H = la^{-n},$$
(15)

where $l > 0$ and $n \geq 0$

Subtracting equation (10) from equation (11), equation (11) from equation (12), equation (10) from equation (12), we have

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{3\ddot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0,$$
(16)

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\left(\frac{\dot{A}}{A}+\frac{2\dot{C}}{C}\right)\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=0\ ,$$

(17)

$$\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}+\left(\frac{\dot{B}}{B}+\frac{2\dot{C}}{C}\right)\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)=0\ .$$

(18)

On solving above equations, we get

$$\frac{B}{A}=d_1\exp\left[c_1\int\frac{dt}{a^5}\right],$$

(19)

$$\frac{C}{B}=d_2\exp\left[c_2\int\frac{dt}{a^5}\right],$$

(20)

$$\frac{A}{C}=d_3\exp\left[c_3\int\frac{dt}{a^5}\right],$$

(21)

where c_1, c_2, c_3 and d_1, d_2, d_3 are constants of integration which satisfy the relation

$$c_1+c_2+c_3=0,\ d_1d_2d_3=1.$$

(22)

Using equation (20), (21) and (22), the metric functions are

$$A=ap_1\exp\left[q_1\int\frac{dt}{a^5}\right],$$

(23)

$$B=ap_2\exp\left[q_2\int\frac{dt}{a^5}\right],$$

(24)

$$C=ap_3\exp\left[q_3\int\frac{dt}{a^5}\right],$$

(25)

Where $p_1=(d_1^{-4}d_2^{-3})^{1/5}, p_2=(d_1d_2^{-3})^{1/5}, p_3=(d_1d_2^2)^{1/5}$

(26)

and $q_1=-\frac{4c_1+3c_2}{5},\ \ q_2=\frac{c_1-3c_2}{5},\ \ q_3=\frac{c_1+2c_2}{5},$

(27)

satisfying the relations

$$p_1p_2p_3^3=1\ ,\quad q_1+q_2+3q_3=0\ .$$

(28)

SOME IMPORTANT PHYSICAL QUANTITIES

In this section we define some important physical quantities.

The average scale factor and the volume scale factor are defined respectively as under

$$a = (ABC^3)^{\frac{1}{5}}, \qquad V = a^5 = ABC^3.$$
(29)

The generalized mean Hubble parameter H is defined by

$$H = (\ln a)_t = \frac{\dot{a}}{a} = \frac{1}{5}[H_1 + H_2 + H_3 + H_4 + H_5],$$
(30)

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = H_4 = H_5 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y, z, u *and* v axes respectively.

The mean anisotropy parameter $\overline{A}$ is given by

$$A = \frac{1}{5}\sum_{i=1}^{5}\left(\frac{\Delta H_i}{H}\right)^2,$$
(31)

where $\Delta H_i = H_i - H$

The expansion scalar θ and shear scalar σ^2 are defined as under

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{3\dot{C}}{C}, \tag{32}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{33}$$

Where $\sigma_{ij} = \frac{1}{2}\left[\nabla_j u_i + \nabla_i u_j\right] - \frac{1}{5}\theta g_{ij},$ (34)

From equation (15) and (30), we have

$$\dot{a} = la^{1-n}, \tag{35}$$

After integrating equation (35), we have

$$a = (nlt + k_1)^{1/n}, \; n \neq 0 \tag{36}$$

and $a = k_2 \exp(lt)$, $n = 0$, (37)

where k_1 and k_2 are constants of integration.

Thus we have two values of the average scale factors which correspond to two different models of the universe.

SIX DIMENSIONAL MODEL OF THE UNIVERSE WHEN $n \neq 0$

In this section we study the five dimensional model of the universe for $n \neq 0$. For this singular model average scale factor *a* given as $a = (nlt + k_1)^{1/n}$ S

The metric coefficients A, B and C turn out to be

$$A = p_1(nlt + k_1)^{1/n} \exp\left[\frac{q_1(nlt + k_1)^{\frac{n-5}{n}}}{l(n-5)}\right], n \neq 5 \tag{38}$$

$$B = p_2(nlt + k_1)^{1/n} \exp\left[\frac{q_2(nlt + k_1)^{\frac{n-5}{n}}}{l(n-5)}\right], \; n \neq 5 \tag{39}$$

$$C = p_3 (nlt + k_1)^{1/n} \exp\left[\frac{q_3 (nlt + k_1)^{\frac{n-5}{n}}}{l(n-5)} \right], n \neq 5 \tag{40}$$

The mean generalized Hubble parameter and the volume scale factor become

$$H = \frac{l}{nlt + k_1}, \qquad V = (nlt + k_1)^{5/n}. \tag{41}$$

The mean anisotropy parameter $\overline{A}$ turns out to be

$$\overline{A} = \frac{q_1^2 + q_2^2 + 3q_3^2}{5l^2 (nlt + k_1)^{(10-2n)/n}}. \tag{42}$$

The deceleration parameter q in cosmology is the measure of the cosmic accelerated expansion of the universe and is defined As

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = n - 1, \quad \text{which is a constant.} \tag{43}$$

A positive sign of q, i.e., $n > 1$ corresponds to the standard decelerating model whereas the negative sign of q, i.e., $0 < n < 1$ indicates inflation. The expansion of the universe at a constant rate corresponds to $q = 0$, i.e, $n = 1$.

The expansion θ and shear scalar σ^2 are given by

$$\theta = \frac{5l}{nlt + k_1} \quad \text{and} \quad \sigma^2 = \frac{q_1^2 + q_2^2 + 3q_3^2}{2(nlt + k_1)^{10/n}}, \tag{44}$$

Thus the energy density of the universe becomes

$$\rho = \frac{1}{15(\lambda + 2\pi)(\lambda + 6\pi)} \begin{bmatrix} (4\lambda + 15\pi)\left\{ \dfrac{10l^2}{(nlt + k_1)^2} + \dfrac{q_1 q_2 + 3q_2 q_3 + 3q_3 q_1 + 3q_3^2}{(nlt + k_1)^{10/n}} \right\} \\ -3\lambda \left\{ \dfrac{5l^2(1-n)}{(nlt + k_1)^2} + \dfrac{q_1^2 + q_2^2 + 3q_3^2}{(nlt + k_1)^{10/n}} \right\} \end{bmatrix}, \tag{45}$$

The pressure of the universe becomes

$$p = \frac{-1}{20(\lambda + 2\pi)(\lambda + 6\pi)} \begin{bmatrix} (\lambda + 12\pi)\left\{ \dfrac{10l^2}{(nlt + k_1)^2} + \dfrac{q_1 q_2 + 3q_2 q_3 + 3q_3 q_1 + 3q_3^2}{(nlt + k_1)^{10/n}} \right\} \\ +4(\lambda + 4\pi) \left\{ \dfrac{5l^2(1-n)}{(nlt + k_1)^2} + \dfrac{q_1^2 + q_2^2 + 3q_3^2}{(nlt + k_1)^{10/n}} \right\} \end{bmatrix}. \tag{46}$$

SIX DIMENSIONAL MODEL OF THE UNIVERSE WHEN $n = 0$

In this section we study the five dimensional model of the universe for $n = 0$.

For this non-singular model average scale factor *a* given as $a = k_2 \exp(lt)$

Here the metric coefficients take the form

$$A = p_1 k_2 \exp(lt) \exp\left[-\frac{q_1 \exp(-5lt)}{5lk_2^5} \right], \tag{47}$$

$$B = p_2 k_2 \exp(lt) \exp\left[-\frac{q_2 \exp(-5lt)}{5lk_2^5} \right], \tag{48}$$

$$C = p_3 k_2 \exp(lt) \exp\left[-\frac{q_3 \exp(-5lt)}{5lk_2^5} \right]. \tag{49}$$

The mean generalized Hubble parameter becomes

$$H = l \tag{50}$$

while the volume scale factor turns out to be

$$V = k_2^5 \exp(5lt). \tag{51}$$

The mean anisotropy parameter $\overline{A}$ becomes

$$\overline{A} = \left[\frac{q_1^2 + q_2^2 + 3q_3^2}{5l^2 k_2^{10}} \right] \exp(-10lt) , \tag{52}$$

while the quantizes θ and σ^2 are given by

$$\theta = 5l , \ \sigma^2 = \left[\frac{q_1^2 + q_2^2 + 3q_3^2}{2k_2^{10}} \right] \exp(-10lt) , \tag{53}$$

Thus the energy density of the universe becomes

$$\rho = \frac{1}{15(\lambda + 2\pi)(\lambda + 6\pi)} \begin{bmatrix} (4\lambda + 15\pi)\left\{ 10l^2 + \dfrac{q_1 q_2 + 3q_2 q_3 + 3q_3 q_1 + 3q_3^2}{k_2^{10} \exp(10lt)} \right\} \\ -3\lambda \left\{ 5l^2 + \dfrac{q_1^2 + q_2^2 + 3q_3^2}{k_2^{10} \exp(10lt)} \right\} \end{bmatrix}, \tag{54}$$

The pressure of the universe becomes

$$p = \frac{-1}{20(\lambda + 2\pi)(\lambda + 6\pi)} \begin{bmatrix} (\lambda + 12\pi)\left\{ 10l^2 + \dfrac{q_1 q_2 + 3q_2 q_3 + 3q_3 q_1 + 3q_3^2}{k_2^{10} \exp(10lt)} \right\} \\ +4(\lambda + 4\pi)\left\{ 5l^2 + \dfrac{q_1^2 + q_2^2 + 3q_3^2}{k_2^{10} \exp(10lt)} \right\} \end{bmatrix}, \tag{55}$$

CONCLUSION

We have obtained two six dimensional exact solutions of Bianchi-I space time in f(R,T) theory of gravity using assumption of constant value of deceleration parameter and variation law of Hubble parameter. The first solution provides a singular model for $n \neq 0$ with power law expansion and second solution gives a non-singular model with exponential expansion for $n = 0$. Cosmological important quantities for both the models are also evaluated.

Six dimensional singular model of the universe with power law expansion for $n \neq 0$ has a singularity at $t = -\frac{k_1}{nl}$. This singularity is point type as metric coefficients vanish at this point.Average scale factor for this model is $a = (nlt + k_1)^{\frac{1}{n}}$. Volume scale factor vanish at this point of singularity. Mean generalized Hubble parameter H, mean anisotropy parameter, expansion scalar, shear scalar are all infinite at this point. This observations indicates that universe starts its expansion with zero volume and continue to expand.

Non -singular model with exponential expansion corresponds to n=0. For this model average scale factor is $a = k_2 \exp(lt)$. Because of exponential behavior, the model is non-singular. Volume of the universe increase exponentially with cosmic time t. Mean Hubble parameter and expansion scalar are constant throughout the evolution. Anisotropy parameter and and shear scalar both are finite for finite values of t. this suggest that expansion takes place with zero volume from infinite past. All the results obtained here are similar to the results obtained by M. Farasat Shamir et al [19].

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